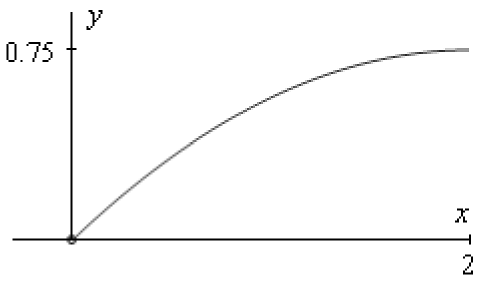
Question			Answer	Marks	Guidance
1	(i)		A Normal test is not appropriate since … … the sample is small and … the population variance is not known (it must be estimated from the data).	 E1 E1 **[2]**	 Allow use of "σ", otherwise insist on "population".
1	(ii)		The sample is taken from a Normal population.	B1 **[1]**	
1	(iii)		H_0: $\mu = 7.8$ H_1: $\mu \neq 7.8$	B1	Both hypotheses. Hypotheses in words only must include "population". Do NOT allow "$\bar{X} = \ldots$" or similar unless $\bar{X}$ is clearly and explicitly stated to be a <u>population</u> mean.
			where μ is the mean water pressure.	B1	For adequate verbal definition. Allow absence of "population" if correct notation μ is used.
			$\bar{x} = 7.631$ $\quad s = 0.1547$	B1	$s_n = 0.1459$ but do <u>NOT</u> allow this here or in construction of test statistic, but ft from there.
			Test statistic is $\dfrac{7.631 - 7.8}{\dfrac{0.1547}{\sqrt{9}}}$	M1	Allow c's $\bar{x}$ and/or s_{n-1}. Allow alternative: 7.8 + (c's –2.896) × $0.1547/\sqrt{9}$ (= 7.65…) for subsequent comparison with $\bar{x}$. (Or $\bar{x}$ – (c's –2.896) × $0.1547/\sqrt{9}$ (= 7.78…) for comparison with 7.8.)
			$= -3.27(7)$.	A1	c.a.o. but ft from here in any case if wrong. Use of $\mu - \bar{x}$ scores M1A0.
			Refer to t_8.	M1	No ft from here if wrong.
			Double-tailed 2% point is ±2.896.	A1	Must compare test statistic with <u>minus</u> 2.896 unless absolute values are being compared. No ft from here if wrong. Allow $P(t < -3.27(7)$ or $t > 3.27(7)) = 0.0113$ for M1A1.
			Significant.	A1	ft only c's test statistic if both M's scored.
			Sufficient evidence to suggest that the mean water pressure has changed.	A1 **[9]**	ft only c's test statistic if both M's scored. Conclusion in context to include "average" o.e.

Question			Answer	Marks	Guidance
1	(iv)		In repeated sampling, 95% of all confidence intervals constructed in this way will contain the true mean.	 E1 E1 **[2]**	
1	(v)		CI is given by 7.631 ± 2·306 $\times \frac{0.1547}{\sqrt{9}}$ = 7.631 ± 0.118(9) = (7.512, 7.750)	M1 B1 M1 A1 **[4]**	ZERO/4 if not same distribution as test. Same wrong distribution scores maximum M1B0M1A0. Recovery to t_8 is OK. Allow c's $\bar{x}$. 2.306 seen. Allow c's s_{n-1}. c.a.o. Must be expressed as an interval.
2	(i)			G1 G1 G1 **[3]**	Curve with positive gradient, through the origin and in the first quadrant only. Correct shape for an inverted parabola ending at maximum point. End point (2, 3/4) labelled.

Question			Answer	Marks	Guidance
2	(ii)		$E(X)=\frac{3}{16}\int_0^2(4x^2-x^3)\mathrm{d}x$	M1	Correct integral for E(X) with limits (which may appear later).
			$=\frac{3}{16}\left[\frac{4x^3}{3}-\frac{x^4}{4}\right]_0^2$	M1	Correctly integrated. Dep on previous M1.
			$=\frac{3}{16}\left\{\left(\frac{32}{3}-\frac{16}{4}\right)-0\right\}$		
			$=\frac{5}{4}$	A1	Limits used correctly to obtain PRINTED ANSWER (BEWARE) convincingly. Condone absence of "–0".
			$E(X^2)=\frac{3}{16}\int_0^2(4x^3-x^4)\mathrm{d}x$	M1	Correct integral for E(X) with limits (which may appear later).
			$=\frac{3}{16}\left[x^4-\frac{x^5}{5}\right]_0^2$	M1	Correctly integrated. Dep on previous M1.
			$=\frac{3}{16}\left\{\left(16-\frac{32}{5}\right)-0\right\}$		
			$=\frac{9}{5}$	A1	Limits used correctly to obtain result. Condone absence of "–0".
			$\mathrm{Var}(X)=\frac{9}{5}-\left(\frac{5}{4}\right)^2=\frac{19}{80}$	M1	Use of Var(X) = E(X^2) – E(X)^2.
			sd = $\sqrt{\frac{19}{80}}$ = 0.487(3)	A1	cao
				[8]	
2	(iii)		$\mathrm{SE}(\bar{X})=\frac{0.487}{\sqrt{100}}$	M1	
			= 0.0487	A1	ft c's σ/10.
				[2]	

Question			Answer	Marks	Guidance
2	**(iv)**		$\mathrm{P}(X<1)=\frac{3}{16}\int_0^1(4x-x^2)\mathrm{d}x$ $=\frac{3}{16}\left[2x^2-\frac{x^3}{3}\right]_0^1$ $=\frac{3}{16}\left\{\left(2-\frac{1}{3}\right)-0\right\}$ $=\frac{5}{16}$	M1 A1 **[2]**	Correct integral for P(X < 1) with limits (which may appear later). cao. Condone absence of "–0" when limits applied.
2	**(v)**		Regard the reed beds as clusters. Select a few clusters (maybe only one) at random. Take a (simple random) sample of reeds (or maybe all of them) from the selected cluster(s).	E1 E1 E1 **[3]**	NB "Clusters of <u>reeds</u>" scores 0 unless clearly and correctly explained.
3			$P1\sim\mathrm{N}(2025,\ 44.6^2)$ $P2\sim\mathrm{N}(1565,\ 21.8^2)$ $I\sim\mathrm{N}(1410,\ 33.8^2)$		When a candidate's answers suggest that (s)he appears to have neglected to use the difference columns of the Normal distribution tables penalise the first occurrence only.
3	**(i)**		$\mathrm{P}(P1<2100)=$ $\mathrm{P}\left(Z<\frac{2100-2025}{44.6}=1.681(6)\right)$ $=0.9536/7$	M1 A1 A1 **[3]**	For standardising. Award once, here or elsewhere. c.a.o.

Question			Answer	Marks	Guidance
3	(ii)		Require P($P1 - P2 > 400$) $P1 - P2 \sim (2025 - 1565 = 460,$ $44.6^2 + 21.8^2 = 2464.4)$ P(this > 400) = $\mathrm{P}\left(Z > \frac{400-460}{\sqrt{2464.4}} = -1.208(6)\right) = 0{\cdot}8864/5$	M1 B1 B1 A1 **[4]**	 Mean. Variance. Accept sd (= 49.64). cao
3	(iii)		$T = P1 + P2 + I \sim \mathrm{N}(5000,$ $\sigma^2 = 44.6^2 + 21.8^2 + 33.8^2 = 3606.84)$ Require b s.t. P($T > b$) = 0.95 $\therefore \frac{b-5000}{\sqrt{3606.84}} = -1.645$ $\therefore b = 5000 - 1.645 \times \sqrt{3606.84} = 4901.2..$	B1 B1 B1 A1 **[4]**	Mean. Variance. Accept sd (= 60.056…). –1.645 seen. c.a.o.
3	(iv)		Mean = $(1.2 \times 2025) + (1.3 \times 1565) +$ $(0.8 \times 1410) = £5592.50$ Var = $(1.2^2 \times 44.6^2) + (1.3^2 \times 21.8^2) +$ $(0.8^2 \times 33.8^2) = 4398.7076 \approx £^2 4399$	B1 M1 A1 **[3]**	Condone absence of £. Use of at least one of $(1.2^2 \times 44.6^2)$ etc… Condone absence of $£^2$.
3	(v)		Mean = (123.72 + 127.38)/2 = 125.55 $s = \frac{127.38 - 125.55}{2.576/\sqrt{50}} = 5.02(3)$	B1 B1 M1 A1 **[4]**	Cao Sight of 2.576. Or equivalent. cao

Question			Answer	Marks	Guidance
4	**(a)**	**(i)**	Number all the projects to be marked. (Sampling frame.)	E1	Do not award if candidate subsequently describes a different method of sampling (eg systematic sampling).
			Use a form of random number generator to select the projects in the sample until 12 projects have been selected.	E1	Condone absence of 12.
				[2]	
4	**(a)**	**(ii)**	H_0: $m = 0$ H_1: $m \neq 0$ where m is the population median difference between the examiners' marks.		This is given in the question.

Diff	15	10	2	–7	11	19	–8	–14	17	13	–5	–4
Rank	10	6	1	4	7	12	5	9	11	8	3	2

Answer	Marks	Guidance
	M1	For differences. ZERO (out of 8) in this section if differences not used.
	M1	For ranks.
	A1	ft from here if ranks wrong.
$W_- = 2 + 3 + 4 + 5 + 9 = 23$	B1	(or $W_+ = 1 + 6 + 7 + 8 + 10 + 11 + 12 = 55$)
Refer to tables of Wilcoxon paired (/single sample) statistic for $n = 12$.	M1	No ft from here if wrong.
Lower (or upper if 55 used) 5% tail is 17 (or 61 if 55 used).	A1	i.e. a 2-tail test. No ft from here if wrong.
Result is not significant.	A1	ft only c's test statistic.
Insufficient evidence to suggest a difference in the marks awarded, on average.	A1	ft only c's test statistic. Conclusion in context to include "average" o.e.
	[8]	

Question			Answer	Marks	Guidance
4	(b)		H_0: The random number function is performing as it should. H_1: The random number function is not performing as it should.	B1	Both hypotheses. Must be the right way round. Allow use of the uniform distribution/model. Do not accept "data fit model" oe.
			All expected frequencies are 10	B1	
			$X^2 = 1.6 + 0.4 + 0.1 + 1.6 + 0.4 + 0.1 + 2.5 + 2.5 + 1.6 + 1.6$	M1	Calculation of X^2.
			$= 12.4$	A1	c.a.o.
			Refer to χ_9^2.	M1	Allow correct df (= cells – 1) from wrongly grouped table and ft. Otherwise, no ft if wrong. $P(X^2 > 12.4) = 0.1916$.
			Upper 10% point is 14.68.	A1	No ft from here if wrong.
			Not significant.	A1	ft only c's test statistic.
			Insufficient evidence to suggest that the random number function is not performing as it should.	A1	ft only c's test statistic. Conclusion in context. Allow in terms of the uniform distribution/model. Do not accept "data fit model" oe.
				[8]	